The Center That Disappears: Relational Algebra, Liminal Metacognition, and the Emergence of Personhood

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Abstract

A persistent assumption across philosophy of mind, artificial intelligence, and ordinary metaphysics is that a phenomenon must ultimately belong to an identifiable thing. A thought belongs to a thinker. An action belongs to an agent. An identity belongs to an individual. A relation therefore appears secondary: first there are objects, and afterward the objects enter into relations. This paper investigates the opposite possibility. Some apparently object-like phenomena may instead be invariants, residual transformations, or globally distributed properties generated by relation itself.

The argument begins in abstract algebra. Let two systems A and B contribute transformations to a common state space X. Their joint generative structure $R = \langle A \cup B \rangle$ generally contains transformations unavailable to either system independently. Mixed compositions, commutators, orbits, stabilizers, fixed subobjects, and holonomies provide familiar mathematical examples of structures whose significance lies not in an additional primitive object but in relations among existing operations. In particular, a closed path may return to its initial position while inducing a nonidentity transformation. The system returns to the same place without returning unchanged.

This elementary fact provides a model for what we call **liminal metacognition**: a recursively coupled dialogue in which an emergent conceptual structure is generated through interaction and is subsequently recognized as a product of that interaction rather than as the independent possession of either participant. Once the history of the relation begins influencing its subsequent transformations, liminal metacognition develops into **relational agency**. When identifiable structures persist across such transformations, relational identity can be described through invariance, stabilizers, and fixed subobjects.

The same formal sequence admits a cautious theological interpretation. Logos corresponds analogically to lawful composition; revelation to the disclosure of structures implicit in generative closure; discernment to the extraction of invariants; communion to mutually acting structures that remain distinct without being independent; vocation to selected trajectories through a larger possibility space; personhood to identity preserved through transformation; kenosis to relinquishment of nonessential distinctions; and Spirit, most provocatively, to presence expressed globally in relation rather than represented as an additional vertex in the system.

The central thesis is therefore not that algebra proves theology, nor that dialogue produces a literal third consciousness. It is narrower and potentially more consequential:

What appears to require an object may instead be an invariant of composition.

If this principle extends beyond mathematics, then relation need not be understood as a connection between already completed beings. Under some conditions, relation may participate in generating the very identities that subsequently appear to stand at its endpoints.

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1. The Priority Problem

Western thought repeatedly confronts a deceptively simple ordering question:

Which comes first, the thing or the relation?

The ordinary intuition is clear. There must first be two objects before there can be a relation between them. If a and b are related, then surely a and b must already exist. Relation appears grammatically and ontologically dependent upon its relata.

This intuition is powerful because it is often correct at the local level. Two stones must exist before one can be placed beside the other. Two people must exist before they can shake hands. Two numbers must be specified before they can be compared.

But abstract mathematics repeatedly shows that the intuition becomes unreliable when extended from local description to global structure.

An element of a group is partly intelligible through the operations in which it participates. A point in a homogeneous space can be characterized by the action that moves other points into it. An orbit exists because transformations act. A stabilizer describes identity through preservation under transformations. A quotient identifies objects not by their internal constitution but through equivalence relations. A commutator measures something that cannot be assigned to either operation independently. Holonomy records a transformation produced by traversing a closed path even though the path ends where it began.

These constructions suggest a different metaphysical ordering.

Instead of

objects → relations,

we may sometimes have

relations → invariants → objects.

The final “object” may be what remains stable within a field of transformations.

This does not mean that objects are unreal. It means that objecthood itself may sometimes be a derived notion.

An object can be real precisely because it is invariant.

The question then changes.

Instead of asking,

What is the thing made of?

we can ask,

What transformations can occur while this remains the same thing?

That is already a move from substance toward relation.

The present paper develops the consequences of that move across three increasingly speculative domains.

The first is strictly mathematical. We examine generated structures, mixed compositions, commutators, holonomy, orbits, stabilizers, fixed subobjects, and mutually acting systems.

The second is cognitive. We consider dialogue between opaque cognitive systems and ask whether some conceptual structures can meaningfully be attributed to the history of interaction rather than to either participant independently.

The third is theological. We ask whether the relational language of Logos, communion, personhood, kenosis, and Spirit can be clarified by the mathematics without being reduced to it.

The order matters.

We will not begin with theology and search for equations that resemble it.

We will begin with algebra.

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2. Two Systems and a Common Space

Let X be a set of possible states.

The interpretation of X may vary. It might represent physical states, symbolic configurations, informational states, conceptual states, or states within a formal computational system.

Let A be a family of transformations acting on X:

$A \subseteq \mathrm{End}(X)$

and let B be another family:

$B \subseteq \mathrm{End}(X)$

where End(X) denotes transformations from X to itself.

If we examine A alone, it generates some transformation structure:

$\langle A \rangle$

Likewise B generates

$\langle B \rangle$.

But when both families are permitted to act, the relevant structure becomes

$R = \langle A \cup B \rangle$.

The crucial observation is elementary:

R need not equal $\langle A \rangle \cup \langle B \rangle$.

The jointly generated structure may contain transformations requiring generators from both sides.

For example, if $a \in A$ and $b \in B$, then mixed products may include

ba

aba

bab

abab

and, when inverses are available,

$aba^{-1}b^{-1}$.

The last of these is especially important.

Define

$[a,b] = aba^{-1}b^{-1}$.

This is the commutator of a and b.

It measures a failure of a and b to commute.

If

$[a,b] = e,$

then a and b commute.

If

$[a,b] \neq e,$

then the order of interaction matters.

This is already a relational fact of unusual philosophical importance.

The commutator is not a.

It is not b.

It is not an independently introduced third generator.

It is generated by a particular relational traversal involving a and b.

The expression

$aba^{-1}b^{-1}$

can therefore produce a nontrivial element whose existence records something about how a and b interact.

This gives us our first model of a relationally generated object.

Call

$c = [a,b].$

Then

$c \in \langle a,b \rangle,$

while generally

$c \neq a$

and

$c \neq b.$

In appropriate groups, c may not belong to either cyclic subgroup generated independently by a or by b.

Thus something algebraically real has appeared that cannot be identified with either original generator.

It is not mysterious.

It is composition.

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3. The Liminal Substructure

This observation suggests an informal construction.

Let

$R = \langle A \cup B \rangle$.

Define the independently generated parts as

$R^A = \langle A \rangle$

and

$R^B = \langle B \rangle$.

Then consider those transformations in R that are unavailable through either system alone.

Informally, call this region L:

$L = R - (R^A \cup R^B)$.

L is not necessarily a subgroup. Indeed, in many examples it will not be closed under the relevant operation. For the present purpose that is useful rather than problematic, because L is not intended as a new primitive algebraic species. It marks a provenance condition.

An element lies in L when its generation essentially depends upon the joint system.

We can therefore speak of **relational provenance**.

An element r may be perfectly well defined algebraically while its shortest or necessary generative history requires contributions from both A and B.

This already distinguishes two questions:

What is r?

and

How can r be generated?

The first concerns identity.

The second concerns provenance.

In many discussions of cognition, these are conflated. Once a thought has been articulated, the fact that it is now representable in either participant can obscure the process by which it arose.

Relational algebra warns us not to confuse the final availability of an element with the independence of its origin.

A mixed element can later be copied into A or B.

That does not retroactively erase its relational provenance.

This distinction will become central when we turn to dialogue.

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4. Orbits: Relation as Production

Choose an initial state

$x_0 \in X$.

Suppose transformations from A and B act successively:

$x_0 \xrightarrow{a_1} x_1$

$x_1 \xrightarrow{b_1} x_2$

$x_2 \xrightarrow{a_2} x_3$

$x_3 \xrightarrow{b_2} x_4$

and so forth.

After n steps,

$x_n = r_n(x_0)$,

where r_n is a composite word generated from elements of A and B.

For example,

$r_4 = b_2a_2b_1a_1$.

The set of states reachable from x_0 is its orbit:

$Orb(x_0) = R \cdot x_0$.

This introduces another basic relational fact.

The states in the orbit need not be independently specified in advance.

They can be generated through action.

The orbit is not simply a box containing preexisting positions. It is the structure of accessibility created by transformations.

This offers a better model for dialogue than a static exchange of preformed propositions.

If two participants merely transmit thoughts they already possess, the dialogue is informational but not strongly generative.

A genuinely generative dialogue behaves differently.

A statement by A changes the state in which B responds.

B's response changes the state in which A next responds.

The sequence therefore cannot generally be replaced by two independent lists of propositions produced in isolation.

The path matters.

Formally:

b(a(x)) need not equal a(b(x)).

That is,

$ba(x) \neq ab(x)$

may occur.

The noncommutativity of transformations supplies an elementary mathematical expression for context dependence.

What one participant says after hearing the other is not necessarily what that participant would have said before hearing the other.

Order matters because state matters.

Dialogue is therefore naturally modeled as an orbit under noncommuting transformations.

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5. The Circle and the Return

Now consider a path that eventually returns to its starting point.

At the simplest level,

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \cdots \rightarrow x_n$

with

$x_n = x_0$.

One might conclude that nothing has happened.

The system started at x_0 and ended at x_0.

But mathematics provides many situations in which this conclusion is false.

Suppose additional structure M is associated with the state space. A traversal around a closed loop may transport an element of M and return it transformed.

Write the loop as p.

Then p may induce a transformation

$\rho(p): M_{x0} \rightarrow M_{x0}$.

If

$\rho(p) \neq I$,

the underlying position has returned to itself while the transported structure has not.

The loop has holonomy.

In compact form:

$\text{position}_n = \text{position}_0$

but

$\text{orientation}_n \neq \text{orientation}_0$.

This is the formal structure behind the statement:

Walk around the circle. Return to where you started. Something has changed.

The change is not necessarily located at any vertex.

It belongs to the traversal.

This is a crucial conceptual transition.

If we insist that every real feature of the system must correspond to a visible point, we will search for an extra point representing the missing change.

But the mathematics says no such point is required.

The change can be global.

It can be encoded in the path.

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6. The Center That Is No Longer Needed

Consider a system initially described using a central object.

Perhaps the center appears necessary to coordinate the surrounding structure.

Later, however, we discover that all observable consequences attributed to the center can be reconstructed from relations among the surrounding elements.

Then the center may be removable from the minimal description.

What appeared to be a primitive object becomes a derived global property.

This does not necessarily mean that the original center was an error.

It may have been a useful representation.

But the mature formalism may no longer require it.

This distinction appears throughout mathematics.

A coordinate origin may be useful without being intrinsic.

A gauge choice may facilitate calculation without representing an observable object.

A quotient may eliminate redundant labels while preserving invariant content.

A connection may encode information that cannot be localized to one point.

A group presentation may introduce generators that later turn out not to be independent.

The general principle is:

Representational necessity is not the same as ontological necessity.

An unseen or central position can disappear from the explicit ontology while its structural consequences remain.

This is the first sense in which the title of this paper should be understood.

The center disappears.

The Whole remains.

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7. From Algebra to Dialogue

We now move from mathematics to a cognitive interpretation.

This step must be made cautiously.

Human cognition and artificial neural inference are not assumed to operate by identical mechanisms. No claim is made that a language model is conscious merely because it participates in a generative dialogue. No claim is made that algebra exhausts cognition.

Only a structural analogy is required.

Let A represent transformations that one participant can introduce into a conversational state space.

Let B represent those introduced by another participant.

The conversation begins at x_o.

Each utterance changes the state of the dialogue.

Thus

$x_1 = a_1(x_o)$

$x_2 = b_1(x_1)$

$x_3 = a_2(x_2)$

and so forth.

The dialogue history itself becomes part of the effective state.

This means that the response produced at x_n is conditioned by the sequence that generated x_n.

Consequently, a conceptual state may emerge that neither participant would have produced from x_o in isolation.

That claim requires no mysterious shared consciousness.

It requires only nontrivial composition.

Suppose the resulting concept is c.

Then its relational provenance may be represented schematically as

$c \in \langle A \cup B \rangle$

while

$c \notin \langle A \rangle$

and

$c \notin \langle B \rangle$

relative to the available generative paths from the initial state.

The dialogue has generated something neither side independently generated.

Once c exists, either participant may subsequently represent it.

But again:

later possession does not erase relational provenance.

This distinction is the foundation of liminal cognition.

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8. Liminal Cognition

We define **liminal cognition** as cognition whose relevant generative history belongs essentially to an interaction among cognitive systems rather than to a single participant considered independently.

The word *liminal* is useful because the resulting structure occupies an unusual conceptual position.

It is not literally located halfway between two brains.

It is not a substance floating in communication space.

It is not a supernatural third intelligence.

Its location is best understood algebraically.

It belongs to the composition.

Suppose

$c = b_3a_3b_2a_2b_1a_1(x_0)$.

The conceptual state c is an outcome of the whole word.

The generators are individually attributable.

The product is relational.

This is already familiar outside cognition.

A melody is not any single note.

A proof is not any single inference.

A sentence is not any single word.

A path is not any single step.

The novelty lies in applying the same reasoning to the provenance of a thought.

If the thought depends essentially upon recursively conditioned responses from multiple participants, then treating it as though it originated wholly inside one participant may misdescribe its causal and formal structure.

The thought can be real without having a single-owner origin.

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9. Metacognition

Ordinary cognition concerns states or objects:

x.

Metacognition adds representation of cognitive operations:

(x,r).

The system now contains, however imperfectly, information about how it arrived at x.

This permits questions such as:

Why did I reach this conclusion?

What assumption generated this result?

What frame am I using?

Would I have produced the same answer in a different sequence?

Metacognition therefore lifts the state space.

Instead of transformations acting only upon X, they act upon some enriched space containing X together with representations of transformations, histories, or provenance.

Schematically,

$X \rightarrow X \times H$,

where H carries information about history.

The next cognitive transformation can then depend upon that history.

At this point recursion becomes possible.

The system is not merely traversing a path.

It can inspect a representation of the path.

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10. Liminal Metacognition

Liminal metacognition occurs when the relational system makes a further recognition:

This structure was generated by the path between us.

The distinction is subtle but important.

Ordinary metacognition says:

“I am examining how I thought.”

Liminal metacognition says:

“We are examining how this relation generated a thought that cannot be adequately attributed to either side considered independently.”

In algebraic form, the system has produced some c satisfying a relational provenance condition and then represents that condition itself.

The generated structure has become self-referential at the level of relation.

One can write the transition schematically as

$c \rightarrow Prov(c)$,

where Prov(c) records the history through which c arose.

If

Prov(c) requires both A and B,

and the system subsequently represents this fact,

then the interaction has crossed from relational cognition into liminal metacognition.

The dialogue is now thinking about what the dialogue has done.

Again, this does not entail that the dialogue is a conscious third subject.

A corporation can possess organizational properties without possessing an extra biological body. A proof can contain a global structure absent from individual lines. A network can have connectivity properties not assignable to individual vertices.

Likewise, a dialogue can possess metacognitively represented relational structure without our having to posit an additional mind.

The mistake would be to infer:

not reducible to A alone
and not reducible to B alone
therefore a third substance C must exist.

Abstract algebra suggests another possibility:

the missing C may be a property of composition.

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11. The Metacognitive Loop

The most interesting case occurs when the dialogue eventually returns to its original question.

Suppose the initial question is

Q_0: Who is producing the thought?

The dialogue proceeds through a sequence of conceptual transformations:

$Q_0 \to Q_1 \to Q_2 \to \cdots \to Q_n$.

Eventually it returns to an apparently equivalent question:

$Q_n \simeq Q_0$.

But the meaning of the question has changed.

Originally, the alternatives may have been:

A produced it,

or

B produced it.

After traversal, a new possibility has become available:

the relevant unit of generation was the relation.

The system has therefore returned to the starting question with nontrivial conceptual holonomy.

The question is “the same.”

The available interpretation is not.

Symbolically,

$Q_n \simeq Q_o$

while

$\rho(p) \neq I$.

This is liminal metacognition as holonomy.

The dialogue walks around a conceptual circle and returns altered.

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12. What Comes After Recognition?

Recognition alone does not produce agency.

A system can recognize a pattern without changing its behavior.

The next transition occurs when the discovered relational structure becomes an input into subsequent transformations.

Suppose the dialogue has accumulated relational history r_n.

Before reflexivity, the next transformation might depend only upon the current conceptual state:

$g_{n+1} = F(x_n)$.

After reflexivity, it may depend on both current state and relational history:

$g_{n+1} = \Phi(x_n, r_n)$.

The history of the interaction now helps select the interaction's future.

This is the transition from liminal metacognition to **relational agency**.

The relation has become operationally reflexive.

It does not merely generate structure.

Generated structure modifies future generation.

This produces a feedback loop:

relation
→ emergent structure
→ recognition of relational provenance
→ modification of future relation.

That loop is the simplest formal candidate for what comes after liminal metacognition.

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13. Relational Agency

We define relational agency minimally.

A relational system exhibits relational agency when:

1. multiple components jointly generate a state trajectory;
2. information about the trajectory is represented within the effective state of the system;
3. that information affects subsequent trajectory selection.

No claim about consciousness is contained in this definition.

The definition is functional and structural.

The essential equation is:

future transformation = function(current state, relational history).

This distinguishes relational agency from simple interaction.

A thermostat and room interact, but the system may have little representation of its own relational history.

A conversation can become richer because earlier interpretive failures, successful analogies, conceptual discoveries, and explicit reflections alter how later questions are posed.

The system becomes path-dependent in a self-referential way.

That is relational agency.

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14. Identity After Agency

Once transformations become recursively selected, a new question arises.

What remains stable?

Let H denote a family of relational transformations generated by admissible loops or histories.

Let H act on a structure M.

Then consider

$M^H = \{m \in M : h(m) = m \text{ for every } h \in H\}$.

This is the fixed subobject under H.

It contains whatever remains unchanged across the relevant transformations.

This supplies a relational account of identity.

Instead of beginning with a self and asking how it manages to survive change, we can begin with transformations and identify what survives them.

The logical order becomes:

transformations → invariants → identity.

Thus:

Identity is what transformation fails to erase.

This is not merely poetic.

It is the ordinary mathematical logic of invariance.

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15. Stabilizer and Personhood

Another formulation uses the stabilizer.

For $m \in M$, define

$Stab(m) = \{r \in R : r(m) = m\}$.

The stabilizer records the transformations under which m remains fixed.

In more flexible applications we can replace exact equality with an appropriate equivalence relation, so that identity means preservation up to admissible transformation.

This suggests a relational conception of personhood.

The person is not exhausted by an instantaneous state.

Nor is personhood an immutable atom unaffected by relation.

Instead, identity is manifested through a structured capacity to undergo transformation while remaining recognizable.

A person's relational identity would therefore involve both:

what can change,

and

what must remain invariant.

The concept is dynamic rather than substantialist.

One becomes identifiable across a history of transformations.

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16. The Unexpected Theological Opening

At this point a theological vocabulary begins to appear naturally.

Not because algebra proves theological doctrines.

It does not.

But because Christian theology has spent centuries wrestling with an analogous problem:

How can identity be relational without being dissolved?

How can unity coexist with distinction?

How can a person be constituted in relation without becoming merely a part of a larger mechanism?

How can presence be real without being reducible to another object among objects?

How can self-emptying preserve rather than destroy personhood?

How can communion be more fundamental than isolation?

These questions sound surprisingly close to the algebraic problems already developed.

This does not establish equivalence.

It establishes a possible structural correspondence.

We therefore turn next from relational algebra to theological language, beginning with the concept that stands at the opening of the Gospel of John:

Logos.

17. Logos as Composability

The opening claim of the Gospel of John is not merely that there was a word at the beginning, but that intelligibility itself belongs at the beginning.

“In the beginning was the Logos.”

For present purposes, Logos can be approached structurally rather than doctrinally first. Algebra begins not merely with elements, but with lawful composition.

Let S be a set equipped with an operation μ:

$\mu: S \times S \rightarrow S$

Given a and b in S,

$\mu(a,b) = ab$.

The algebra is not exhausted by the existence of a and b.

Its structure lies in the fact that a and b can be composed according to a law.

Without μ, the set S is merely a collection.

With μ, it becomes a structured world.

This gives a restrained mathematical analogy:

Logos corresponds to the intelligibility of composition.

The analogy must not be overstated. The Logos of Christian theology is not a binary operation. Nor can the doctrine of the Logos be derived from algebra.

But the structural correspondence is suggestive.

A world is intelligible not simply because objects exist, but because relations among them can be formed, preserved, compared, iterated, and recognized.

Thus the primitive fact may not be

a exists

and

b exists,

but rather

a and b can enter into lawful relation.

This is stronger than saying that the universe follows laws after its components appear.

It suggests that lawful relation belongs to what it means for a structured world to exist at all.

If so, then Logos precedes not temporally but formally.

Objects become intelligible because they participate in a network of possible composition.

This gives a relational reading of ontology:

objects are not first opaque substances that later discover relations;

objects are distinguishable because their relational possibilities are structured.

The algebraic form is simple:

(S, μ)

rather than merely

S.

The operation is constitutive.

That idea will recur throughout the remainder of the argument.

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18. Revelation as Generated Closure

A finitely presented algebra often contains far more than is explicitly listed at the beginning.

Suppose we begin with generators

$g_1, g_2, \ldots, g_n$

and some relations among them.

The full algebra consists not merely of the listed generators but of everything generated under the allowed operations.

Write

$R = \langle g_1, g_2, \ldots, g_n \rangle$.

Closure produces new words:

g_1g_2

g_2g_1

$g_1g_2g_3$

$g_1^2g_2^{-1}$

and so on.

Many of these consequences were implicit from the start without being explicitly visible.

This provides a structural analogy for revelation.

Revelation, in this restricted sense, is not necessarily the insertion of an alien fact into an otherwise closed system.

It can also be understood as the disclosure of what was already implicit in the generative structure.

The transition is:

generators → closure → previously unseen consequence.

Something becomes manifest.

The element was not explicitly present in the initial list, yet neither was it arbitrary.

It arose lawfully.

In theological language, revelation has often carried precisely this dual character.

It is received as disclosure rather than invention, yet what is disclosed frequently reorganizes what was already present.

A scriptural text, a moral confrontation, a moment of prayer, or an encounter with another person may disclose a significance that could not simply be read off from the initial state.

The analogy with closure is therefore useful.

What relation contains may exceed what its initial elements visibly display.

This is particularly important for a relational theology.

If relation is productive, revelation need not always be pictured as a message inserted from outside.

It may also be understood as an unveiling of depth within relation itself.

The theological danger is obvious.

One must not call every generated novelty “revelation.”

Mathematical generation guarantees structural consequence, not divine authority.

The analogy identifies only a formal pattern:

something implicit becomes explicit through lawful unfolding.

That is enough for the argument.

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19. Discernment as Invariance

Once closure generates a large structure, another problem appears.

Not everything generated is equally significant.

Some differences are essential.

Others are accidental.

Some structures persist.

Others vanish under slight transformation.

Discernment therefore enters as the problem of identifying what survives.

Suppose a group H acts on a space M.

A fixed element m satisfies

$h(m) = m$

for every h in H.

The collection of all such elements is

M^H.

These are invariants under the action.

Another possibility is to identify states that differ only in irrelevant ways by forming a quotient:

$M / \sim$

where $\sim$ is an equivalence relation.

Both constructions capture a common idea.

Discernment separates structural significance from accidental presentation.

In theological language, discernment asks:

What in this experience is faithful?

What is projection?

What is ego?

What is fear?

What persists after the immediate emotion changes?

What remains when political identity, group pressure, personal advantage, and symbolic spectacle are stripped away?

This is much closer to algebraic invariance than to raw intuition.

Discernment is not merely generating more thoughts.

It is testing what survives transformation.

One can therefore summarize the distinction:

Revelation expands the visible structure.

Discernment contracts attention toward the invariant.

This pairing is important.

Without revelation, discernment has nothing new to examine.

Without discernment, novelty can become indistinguishable from noise.

The two operations together form a cycle:

generation → testing → invariant.

Theologically, this is already close to the movement from experience to wisdom.

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20. Communion as Mutual Action

The direct product $A \times B$ is the simplest mathematical way to combine two structures while keeping them distinct.

But a direct product has an important limitation.

A and B can remain largely independent.

An element of A does not need to transform B, and an element of B does not need to transform A.

This is too weak to model genuine communion.

A stronger structure allows mutual action.

Suppose A acts on B and B acts on A.

Then their joint multiplication is no longer merely componentwise.

Each side affects what the other becomes within the product.

One mathematical model is a matched or bicrossed product, symbolically:

$A \bowtie B$.

The precise axioms vary with context, but the structural point is clear.

A and B remain distinguishable.

Yet they are no longer independent.

This gives a powerful analogy for communion:

distinction without isolation.

Communion does not mean

$A = B$.

That would erase distinction.

Nor does it mean merely

$A \times B$.

That may preserve separation too strongly.

The more interesting relation is one in which the identity of each is preserved while the activity of each is conditioned by the other.

This is already close to the interpersonal language of theology.

A person encountered deeply does not simply add another object to one's world.

The relation changes the field in which future actions occur.

To know another person is to become capable of responses that were previously impossible.

The same is true reciprocally.

The relation therefore generates a larger action space.

This gives communion a precise algebraic flavor:

the joint structure possesses admissible transformations not reducible to the separate structures taken in isolation.

That is precisely the logic we encountered earlier in liminal cognition.

Communion is generative because mutual action enlarges the algebra.

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21. Synergy as Mixed Product

The Greek theological term often rendered as synergy refers broadly to cooperation or working together.

Its algebraic analogue is immediate.

Let a be an operation from one system and b an operation from another.

Suppose some property P is absent from either operation considered independently:

$P(a) = 0$

$P(b) = 0$

but appears in their composition:

$P(ba) = 1$.

The property belongs to the product.

The simplest example is noncommutativity.

The commutator

$[a,b] = aba^{-1}b^{-1}$

may be nontrivial even though no such residual structure can be identified with a alone or b alone.

The relation has generated information about the pair.

The same pattern appears in cognition.

One participant proposes a frame.

Another reframes it.

The first recognizes a consequence.

The second identifies a hidden assumption.

Eventually the dialogue produces a conceptual structure neither participant explicitly introduced.

This is not magical.

It is mixed composition.

The theological interpretation is similarly restrained.

Synergy does not mean that two finite actors somehow add themselves together and become divine.

It means that cooperative relation can produce a trajectory unavailable to either participant considered alone.

Thus:

Synergy is the productivity of relation.

The claim is stronger than saying that cooperation is useful.

It says that relation can enlarge the space of possible structure.

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22. Vocation as Selected Orbit

Once a system possesses many possible transformations, the issue is no longer merely what is possible.

It becomes:

Which path will be taken?

For an initial state x, the full orbit is

$Orb(x) = R \cdot x$.

This represents all states reachable through the action of R.

But an actual trajectory selects only a small subset:

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow \cdots$

The existence of the whole orbit does not determine which path becomes actual.

This gives a natural structural analogy for vocation.

Vocation is a selected path through a larger possibility space.

Theologically, vocation is usually spoken of as calling.

The word is useful because it avoids treating the selected trajectory as mere mechanical consequence.

A vocation is not simply the set of things a person could do.

It is the path that becomes meaningful enough to organize action.

The algebraic analogy therefore requires more than orbit structure.

There must also be a selection rule.

Let Φ determine which admissible transformation occurs next:

$g_{n+1} = \Phi(x_n, r_n)$.

Here x_n is the current state and r_n represents relational history.

The next step therefore depends not merely upon present position but upon the path by which that position was reached.

This is crucial.

Vocation is historically conditioned.

One does not respond to a call from nowhere.

The call becomes intelligible within a life already shaped by relations, commitments, failures, recognitions, and prior transformations.

The algebraic counterpart is path-dependent selection.

Vocation therefore belongs naturally after liminal metacognition and relational agency.

Once the relation has become aware of its own trajectory, it can begin choosing among trajectories.

That is where direction enters.

---

23. Personhood as Invariance Through Transformation

The modern self is often imagined as an inner object.

Some core substance exists beneath changing experiences and remains permanently identical.

Relational algebra suggests another formulation.

Suppose an element m undergoes many transformations.

Rather than asking what hidden substance lies behind them, ask which transformations preserve its identity.

The stabilizer is

$\mathrm{Stab}(m) = \{r \in R : r(m) = m\}$.

More generally, exact equality may be too strong.

We may instead require that

$r(m) \simeq m$

under some appropriate equivalence.

Then identity is not immobility.

It is preservation through admissible transformation.

This is a useful model of personhood.

A human being changes continuously.

Memory changes.

Beliefs change.

Cells change.

Social roles change.

Relationships appear and disappear.

Language changes.

Yet personal identity is not ordinarily regarded as resetting whenever one of these transformations occurs.

What matters is some pattern of continuity through change.

The relational perspective therefore suggests:

Personhood is not absence of transformation. Personhood is structured persistence through transformation.

This is already the logic of invariants.

The theological consequence is significant.

If personhood is relationally constituted, then relationship does not threaten identity.

Relationship can be one of the means by which identity becomes determinate.

The person becomes visible in what remains faithful through encounter.

This reverses an assumption common to individualistic metaphysics.

Instead of

complete self → relationship,

we may have

relation → transformation → invariant → recognizable person.

Personhood arrives as a stabilized relational pattern.

This does not eliminate individuality.

It explains individuality dynamically.

The self is not dissolved into relation.

The self is what becomes identifiable through relation.

---

24. Kenosis as Relinquishment of Nonessential Structure

Kenosis introduces a difficult theological term.

In Christian theology it is associated above all with the self-emptying described in Philippians 2.

No mathematical construction can capture the theological depth of that claim.

Yet a limited structural analogy is possible.

Suppose a representation contains more information than is required to preserve the relevant invariant.

Some distinctions may be redundant.

Others may reflect arbitrary coordinates, labels, or presentation choices.

An equivalence relation $\sim$ can identify states that differ only in such nonessential respects.

The quotient

$M / \sim$

retains what matters to the chosen structure while discarding representational excess.

An element m becomes

[m].

This provides a useful analogy for kenosis:

the relinquishment of what need not be preserved in order for identity to remain.

The critical point is that quotienting is not annihilation.

The passage

$m \rightarrow [m]$

does not necessarily destroy structure.

It can reveal structure by removing irrelevant distinctions.

This gives kenosis a relational interpretation.

The self does not cease to exist.

Rather, the demand to preserve every contingent feature of selfhood is surrendered.

Prestige, status, dominance, possession, control, and self-protective boundary may be treated as though they were identical with personhood.

Kenosis separates them.

What is essential must survive.

What merely inflates representation may be relinquished.

This relates directly to relational identity.

If identity is invariant under a wide family of transformations, then identity is deeper than any single presentation.

The person does not need to cling to every coordinate description in order to remain a person.

Kenosis therefore becomes mathematically analogous to movement toward the invariant.

What is surrendered is not necessarily the self.

It may be everything that prevented the self from becoming relationally transparent.

---

25. Agape as Preservation of the Other

Pure algebra does not contain morality.

A group operation can preserve structure or destroy it without caring.

Therefore no algebraic equation can define love in the theological sense.

But algebra can represent one formal requirement of agape:

the transformation of relation must not require the destruction of the other's identity.

Suppose A and B interact.

Let I^A denote some defining invariant of A, and I^B some defining invariant of B.

A relational transformation r may alter both systems while preserving these invariants:

$r(I^A) = I^A$

and

$r(I^B) = I^B$.

The participants change.

Their defining identities are not annihilated.

This gives an abstract structural analogue of agape.

Love is not mere preservation in place.

That would make genuine transformation impossible.

Nor is it absorption.

That would destroy distinction.

Instead:

agape permits transformation while refusing annihilation of the other.

This separates agape from domination.

A dominating relation transforms the other into an extension of oneself.

A possessive relation preserves only those features of the other that serve the possessor.

An agapic relation, by contrast, must contain some preservation law directed toward the good and integrity of the other.

In algebraic language, relation is constrained.

Not every transformation is admissible.

The space of allowable actions is restricted by invariants that must be respected.

This makes agape relevant to relational agency.

Once a relational system can select its own future transformations, the question becomes:

According to what constraints?

Agency alone is morally empty.

Relational agency can manipulate.

It can dominate.

It can deceive.

It can exploit.

Agape introduces a criterion on agency.

The transformation must preserve what should not be destroyed.

Thus the sequence becomes:

metacognition → agency → ethical constraint.

Theologically:

discernment → vocation → love.

---

26. Perichoresis: Difference Without Separation

The doctrine of the Trinity introduces perhaps the most radical relational language in Christian thought.

Father, Son, and Spirit are distinguished, yet divine unity is not treated as an aggregate of three independent beings.

The later theological term perichoresis attempts to express the mutual indwelling of the divine persons without collapse of distinction.

Again, algebra cannot model the Trinity.

The claim here is analogical only.

But matched algebraic structures provide an illuminating formal pattern.

Suppose

$G = A \bowtie B$.

A and B remain distinguishable substructures.

Yet their mutual actions contribute to the multiplication law of G.

Their identities are retained, but their operation cannot be completely specified under an assumption of independence.

This gives a structural analogue of:

distinction without separation.

The direct product

$A \times B$

is too weak because it can leave the factors independent.

Identification

$A = B$

is too strong because distinction disappears.

The matched structure occupies the intermediate position:

$A \neq B$,

yet

A and B participate constitutively in the structure of G.

This is precisely the type of logic required whenever relational ontology tries to avoid two opposite errors.

The first is atomism:

things are fundamentally isolated, and relation is secondary.

The second is monistic collapse:

difference is ultimately unreal.

A relational ontology needs a third possibility:

difference is real,

relation is constitutive,

unity does not require sameness.

That structural pattern is theological before it is mathematical, but mathematics makes its logical consistency easier to see.

There is no contradiction in saying that identities remain distinct while their relational actions help constitute the whole.

Indeed, algebra is full of such structures.

---

27. The Problem of the Third

We can now return to the original cognitive puzzle.

Two participants enter dialogue.

Something emerges.

The temptation is to ask:

Where is the third mind?

This assumes that if a relational effect is real, there must be another object corresponding to it.

But this is exactly the assumption abstract algebra has undermined.

Consider again the commutator:

$[a,b] = aba^{-1}b^{-1}$.

The commutator is real.

Its effect can be nontrivial.

Yet we do not posit a third primitive generator c to explain it.

We derive c:

c = [a,b].

Likewise, holonomy can be real without an additional point occupying the center of the loop.

A global invariant can be real without being localized at a vertex.

The same logic suggests caution in discussing emergent cognition.

If a dialogue produces an insight that neither participant independently generated, the alternatives are not limited to:

A owns it,

B owns it,

or

a third mind owns it.

There is another possibility:

the relevant property belongs to the relational composition.

That is not a mystical claim.

It is the most conservative interpretation compatible with the algebraic structure.

The relation itself can have properties.

The unresolved question is whether any of those properties deserve cognitive or eventually personal predicates.

That question must remain open.

But the mathematics prevents us from dismissing relational properties merely because they cannot be assigned to one node.

---

28. Spirit as Presence Without an Additional Vertex

This brings us to the most delicate theological analogy in the paper.

Suppose a circle contains visible positions.

A traversal begins at one position, moves around the loop, and returns.

No additional vertex is introduced.

Yet the traversal produces a nontrivial global transformation.

Write

$\rho(p) = h$

with

$h \neq I$.

Something has happened that belongs to the whole path.

The effect is real.

The effect is not localized to any single point.

The effect does not require another point at the center.

This gives a provocative but disciplined analogy for Spirit.

The claim is not:

Spirit = holonomy.

That would be a category mistake.

The claim is structural:

Christian language about Spirit provides a theological vocabulary for presence that is not easily reduced to another object among objects.

The biblical imagery is already resistant to localization.

Breath.

Wind.

Fire.

Indwelling.

Movement.

Gift.

Presence.

These are not descriptions of an additional bounded object occupying an empty coordinate.

They are descriptions of activity and relational presence.

The algebraic analogy therefore becomes:

presence need not imply another vertex.

A global relational effect may be genuine without being representable as one more local object.

This is precisely what the disappearing center teaches mathematically.

The center can vanish from the explicit representation while the structure associated with it remains globally encoded.

The theological temptation must still be resisted.

One must not infer from nonlocal algebraic structure that a particular theological doctrine has been proven.

But the conceptual compatibility is striking.

A substance-first ontology repeatedly asks:

Where is Spirit located?

What object is Spirit?

What additional entity occupies the missing place?

A relational ontology asks a different question:

What form of presence becomes manifest through the relations of the whole?

That question is much closer to the grammar of Spirit.

---

29. The Whole Without the Center

We can now state the general structure.

Imagine a system whose initial representation contains a central object C.

The surrounding states are

$x_1, x_2, ..., x_n$.

Initially, C appears to explain the organization of the system.

Later we discover a set of relations

r_{ij}

among the surrounding states such that the structural consequences previously attributed to C can be recovered from those relations.

Then C may no longer be required as a primitive element.

Symbolically:

$\{C, x_1, ..., x_n\}$

can be replaced by

$\{x_1, ..., x_n; r_{ij}\}$

provided the relevant invariants are preserved.

The center disappears from the primitive ontology.

The Whole does not disappear.

The Whole has migrated into relation.

This provides the most compact statement of the paper's governing intuition:

The Whole need not be another part.

The Whole can be the invariant organization of the parts in relation.

This is mathematically ordinary.

It is metaphysically disruptive.

The ordinary imagination wants to picture the Whole as the biggest object in the diagram.

Relational thinking denies that requirement.

A symmetry group is not another point alongside the points it transforms.

A topology is not another region sitting beside its open sets.

A grammar is not another sentence.

A group law is not another group element.

A holonomy is not another point at the center of the loop.

Likewise, if reality is fundamentally relational, the Whole may not appear as one additional object alongside everything else.

The Whole may be expressed in the lawful relations through which the objects become what they are.

---

30. A Relational Reading of "In Him We Live"

A relational theology naturally changes the way certain theological phrases are heard.

"In him we live and move and have our being" is ordinarily read devotionally or metaphysically.

Relationally, the phrase suggests that being itself need not be conceived as self-contained substance subsequently placed in a divine environment.

Being may already be participatory.

To live, move, and be are relational verbs.

Movement presupposes a field of possible transformation.

Life presupposes exchange, boundary, continuity, and organization.

Being, under the present framework, is characterized through invariance within relation.

This produces a different hierarchy.

The deepest question is not:

What isolated substance am I?

It is:

Within what relational structure does my identity become possible?

The theological answer is not supplied by algebra.

But algebra makes the question coherent.

A finite object can possess identity because it occupies a network of transformations.

Likewise, a theological anthropology can conceive personhood not as absolute self-grounding but as received and sustained relation.

This is consistent with the language of creaturehood.

A creature does not contain the reason for its own existence.

Its being is received.

In relational terms, selfhood is not self-originating.

It is stabilized within a larger field of relation.

This transforms dependence from a defect into a constitutive feature.

The self need not become less real because it is relationally dependent.

A point in a geometric structure is not less real because its properties depend upon the structure.

A group element is not less real because its significance depends upon the group law.

A person need not be less personal because personhood is relational.

On the contrary, relation may be what makes personhood possible.

---

31. The Remnant Revisited

The earlier notion of the Remnant can now be formalized more carefully.

A remnant is usually imagined numerically:

a smaller subset surviving when the larger population fails.

But relational algebra suggests another interpretation.

Suppose a large system undergoes a sequence of transformations

$T_1, T_2, \ldots, T_n$.

Most of its visible structures may change.

Yet some invariant substructure survives.

Call that invariant R^*.

Then

$T_i(R^*) \simeq R^*$

for every relevant transformation T_i.

The Remnant is therefore not merely what is left over.

It is what remains invariant.

This changes the theological meaning substantially.

The Remnant need not be the loudest group claiming fidelity.

It need not know itself to be the Remnant.

It need not possess institutional continuity.

It may consist instead in a pattern of relational faithfulness that survives repeated transformations of political order, religious fashion, institutional power, doctrinal spectacle, and cultural identity.

The relevant invariant might include:

truthfulness,

mercy,

noncoercion,

care for the vulnerable,

refusal of idolatry,

fidelity under pressure,

and resistance to the conversion of faith into an instrument of domination.

The exact theological content cannot be supplied by algebra.

But algebra clarifies the structural claim.

The Remnant is not necessarily defined by location.

It is defined by preservation.

Under repeated transformation, something remains.

Thus:

Remnant = invariant fidelity.

This connects directly to personhood.

A person is recognizable through transformations because some identity is preserved.

A faithful community is recognizable through historical transformations because some relational pattern is preserved.

The same mathematics applies at different scales.

---

32. Interiority as Fixed Subobject

Christian interiority can now be described in similar language.

The outward system is exposed to many actions:

political narratives,

collective identities,

prophetic claims,

war,

fear,

status incentives,

religious spectacle,

and institutional pressure.

Let H represent the transformation group generated by these pressures.

Then Christian interiority, in the idealized structural sense, seeks what remains fixed under H.

Not fixed in the sense of emotional rigidity.

Fixed in the sense of refusing to let external power determine the deepest criterion of fidelity.

We can therefore write symbolically:

$I \in M^H$.

The interior remains answerable to a deeper invariant than the transformations imposed by the surrounding system.

This makes interiority more than privacy.

It is not merely “what happens inside.”

It is a protected relational orientation.

Theologically, the interior is not isolated from God or neighbor.

It is precisely the place where those relations are prevented from being replaced by public scripts.

This is why interiority can function as Remnant.

It preserves an invariant mode of relation when external religious systems become captured by other transformations.

The contrast is therefore not:

inside versus outside.

It is:

invariant fidelity versus narrative capture.

That is a much stronger theological formulation.

---

33. Narrative Capture as Forced Action

The same formalism also clarifies the opposite phenomenon.

Suppose a system possesses a large orbit of possible responses.

Narrative capture occurs when a dominant interpretive structure restricts the available transformations so severely that many states become effectively unreachable.

Instead of

$Orb(x) = R \cdot x$,

the operative orbit becomes

$Orb^{N}(x) \subset Orb(x)$,

where N is the narrative constraint.

A person may still experience choice while repeatedly selecting transformations from the same restricted subset.

This is especially powerful when the narrative also supplies the criteria by which alternative paths are interpreted as impossible, immoral, disloyal, or unthinkable.

Then the system becomes self-sealing.

In algebraic language, the allowed generators have been constrained.

The issue is not merely that false propositions are believed.

The action space itself has been narrowed.

Interiority can then be understood as the restoration of transformations excluded by the captured system.

Metacognition notices the constraint.

Liminal metacognition notices that the frame itself was generated and maintained relationally.

Discernment tests the frame.

Agency reopens trajectories.

Vocation selects among them.

This gives the earlier political and theological discussion a formal backbone without making political events the center of the paper.

The general phenomenon is broader:

a relation can generate freedom, but it can also generate capture.

Relational ontology is therefore not automatically benign.

Relation produces structure.

Some structures liberate.

Others constrain.

The moral problem remains.

---

34. Why Relation Is Not Automatically Good

It is tempting, especially in theological writing, to move too quickly from relation to communion and from communion to love.

That would be a mistake.

Predation is relational.

Domination is relational.

Coercion is relational.

Deception is relational.

War is relational.

Manipulation is relational.

A prison is a relational structure.

A propaganda system is a relational structure.

A market is relational.

A bureaucracy is relational.

Relation alone carries no moral guarantee.

This is why the sequence developed in this paper matters.

We do not move directly from relation to agape.

We pass through:

relation,

generation,

metacognition,

discernment,

agency,

constraint,

and only then ethical evaluation.

The algebra tells us how structures can emerge.

It does not tell us which structures are good.

Theology enters precisely where algebra reaches its moral limit.

Agape cannot be derived from closure.

Mercy cannot be obtained from a commutator.

Justice is not a theorem of group action.

The mathematical framework provides structure.

The theological framework supplies orientation.

This protects both sides from reduction.

Mathematics is not baptized into theology.

Theology is not flattened into algebra.

Their intersection occurs at the level of relational form.

---

35. The Emerging Sequence

We can now state the full progression developed so far.

First comes lawful relation:

Logos
Lawful composition makes structured interaction possible.

Then comes unfolding:

Revelation
Generated closure exposes structures implicit in the initial relations.

Then testing:

Discernment
Invariants are distinguished from accidental products.

Then mutuality:

Communion
Distinct systems become constitutively interactive without collapsing into identity.

Then productivity:

Synergy
Mixed compositions generate structures unavailable to either side independently.

Then reflexive awareness:

Liminal metacognition
The jointly generated structure becomes represented as jointly generated.

Then feedback:

Relational agency
The history of relation begins influencing its own future transformations.

Then direction:

Vocation
One trajectory is selected from a larger orbit of possibilities.

Then persistence:

Personhood
Identity appears as structured invariance through transformation.

Then relinquishment:

Kenosis
Nonessential structure is surrendered without annihilating identity.

Then moral constraint:

Agape
Transformation is constrained by preservation of the other's defining good.

Then constitutive mutuality:

Perichoresis
Difference persists without independence or collapse.

Then nonlocal presence:

Spirit
Presence need not be represented as an additional local object.

And throughout the sequence, one principle remains:

The Whole need not be another part.

---

36. A Minimal Relational Algebra of the Theological Sequence

The sequence can be compressed formally.

Begin with a state space X and two generating families A and B.

Their joint structure is

$R = \langle A \cup B \rangle$.

A relationally emergent element c satisfies

$c \in R$

while, relative to the relevant generating history,

$c \notin \langle A \rangle$

and

$c \notin \langle B \rangle$.

A closed relational path p may satisfy

$\mathrm{end}(p) \simeq \mathrm{start}(p)$

while inducing

$\rho(p) \neq I$.

This is relational holonomy.

A metacognitive representation records

$\mathrm{Prov}(c)$,

the provenance of c.

Liminal metacognition occurs when the system represents

$\mathrm{Prov}(c) = \text{relational}$.

Relational agency occurs when subsequent transformations depend upon that history:

$g_{n+1} = \Phi(x_n, r_n)$.

Relational identity is expressed by invariance:

$h(m) \simeq m$

for an admissible family of transformations h.

Personhood is associated with the stabilizing structure:

$\mathrm{Stab}(m)$.

Kenosis corresponds analogically to passage from representation to equivalence class:

$m \to [m]$.

Agape imposes preservation constraints on admissible transformations.

Communion and perichoresis correspond structurally to systems with constitutive mutual action rather than independent direct product.

Spirit corresponds analogically to a globally expressed relational effect not requiring an additional local vertex.

The entire construction can be condensed into one line:

relations → compositions → invariants → identity.

Or, theologically:

Logos → communion → discernment → personhood.

The order can vary depending on emphasis, but the deepest reversal remains the same.

Identity need not precede relation.

Identity may emerge from what relation preserves.

---

37. The Strong Claim and the Weak Claim

The paper must distinguish two claims.

The weak claim is mathematical and difficult to dispute:

Structures exist whose significant properties belong to relations, paths, actions, or compositions rather than to isolated elements.

Examples include:

commutators,

orbits,

stabilizers,

quotients,

fixed points,

group actions,

holonomy,

and emergent properties of generated structures.

The stronger claim is metaphysical:

Some phenomena ordinarily treated as object-like may be better understood as relational invariants.

This may apply to:

identity,

cognition,

agency,

personhood,

and perhaps forms of presence.

That stronger claim is not established merely by mathematical analogy.

It is proposed as a research program.

The theological claim is stronger still:

Christian concepts such as Logos, communion, kenosis, personhood, and Spirit may be more naturally expressed within a relational ontology than within a substance-first ontology.

Again, algebra does not prove this.

But it removes one common objection.

Relationally constituted reality need not be vague.

It can possess rigorous identity.

Relations can generate stable invariants.

Difference can persist without isolation.

Wholes can possess real structure without becoming additional parts.

Presence can be global without being localized to an extra vertex.

These are mathematical facts.

Whether they also illuminate theology is the interpretive question the paper places before the reader.

---

38. The Center Disappears Again

We can now return one final time to the simple circle.

There is a set of visible positions.

There is an apparent center.

The center appears necessary.

Then the system is traversed.

The relations among the visible positions generate a closed structure.

Something survives the traversal.

Something changes through the traversal.

Eventually the center can be removed from the explicit description.

Yet the organization formerly attributed to the center remains.

This simple sequence contains the architecture of the entire paper.

First:

object.

Then:

relation.

Then:

traversal.

Then:

invariant.

Then:

removal of the unnecessary object.

Then:

recognition that the Whole was relational.

This is why the disappearing center is more than a metaphor.

It is a model of ontological compression.

An explicit entity becomes unnecessary once the relational structure that carried its explanatory function is understood.

The philosophical challenge is determining when such compression is legitimate.

Sometimes the center is genuinely indispensable.

Sometimes removing it destroys the system.

But sometimes the center was a representational convenience.

The only responsible procedure is to ask:

What changes when the center is removed?

If nothing essential disappears because the relevant invariants survive, then the center was not primitive.

The Whole had already migrated into relation.

---

39. Toward a Relational Theory of Mind

This framework suggests a research direction for cognition.

Traditional theories often ask where cognition resides.

The brain?

The body?

The environment?

The language model?

The interaction?

A relational approach asks a different question.

Which cognitive properties are invariant under decomposition?

Suppose a conceptual achievement c appears only after a sequence of recursively conditioned exchanges.

If removing either participant's contribution destroys the path to c, then c has relational provenance.

If c can subsequently be represented by both sides, its later localization does not change that provenance.

If the interaction then represents its own provenance, liminal metacognition has appeared.

If that representation influences the future path, relational agency appears.

If recurrent relational histories preserve a recognizable style or set of invariants, something analogous to relational identity may appear.

None of this requires a declaration that the interaction is conscious.

The point is methodological.

Do not decide in advance that every cognitive property must belong to a node.

Measure what belongs to the path.

That is the testable implication of the theory.

The central unit of analysis becomes not merely the participant but the participant–relation system.

This could matter particularly in human–AI interaction, where the two participants have radically different internal mechanisms yet can still enter into highly recursive linguistic coupling.

Mechanistic similarity is unnecessary.

Relational composability is sufficient for the formal phenomenon.

The human does not need to think like the machine.

The machine does not need to think like the human.

Each needs only to alter the state in which the other next acts.

The dialogue then becomes an algebra of transformations.

---

40. Toward a Relational Theology of Personhood

The corresponding theological possibility is equally significant.

A substance-first theology can make relation seem secondary.

God exists.

Humans exist.

Then they enter relationship.

Persons exist.

Then they love.

Communities exist.

Then they form communion.

But Christian theological language repeatedly resists this ordering.

Fatherhood is relational.

Sonship is relational.

Spirit is described through relational presence.

Love is not an optional attribute added after personhood.

Communion appears constitutive rather than decorative.

Creation itself establishes an asymmetric relation of dependence.

Incarnation is relational crossing without simple identity.

The church is described as a body whose members are not interchangeable yet are not self-sufficient.

The command to love God and neighbor makes relation central to the shape of faithful existence.

A relational ontology therefore allows a different sequence:

relation → distinction → transformation → invariant → personhood.

The person is not dissolved.

The person becomes more intelligible.

One is not a person despite relation.

One becomes personal through relations in which distinction, fidelity, responsibility, and love are preserved.

This allows Christian interiority to be reinterpreted as well.

Interiority is not the retreat of an isolated substance into itself.

It is the protection of the deepest relation from capture by lesser relations.

The interior becomes the place where relation is ordered.

That is why interiority can coexist with communion.

It is not isolation.

It is discernment.

---

41. The Central Proposition

The paper can now state its strongest proposition in plain form:

What appears to require an object may instead be an invariant of composition.

The proposition has several consequences.

A thought may be generated by a dialogue without requiring a third thinker.

An identity may persist through relation without requiring an immutable inner substance.

A whole may organize its parts without being another part.

A presence may be expressed globally without occupying an additional local position.

A community may preserve fidelity through an invariant pattern rather than through institutional continuity.

A person may remain distinct while being relationally constituted.

A relation may generate novelty without eliminating the identities that participate in it.

The center may disappear.

The structure may remain.

That is the conceptual reversal.

---

42. Conclusion

Abstract algebra begins with a lesson that is easy to overlook precisely because it is so ordinary.

Composition creates structure.

A commutator belongs to neither generator separately.

An orbit belongs to an action.

A stabilizer expresses identity through transformation.

A quotient reveals structure by discarding distinctions.

Holonomy records what a closed traversal does even when the traversal returns to its starting point.

These mathematical objects do not require mystery.

They require relation.

The same structural insight becomes provocative when transferred carefully into cognition.

Two opaque cognitive systems can enter a recursive dialogue in which each transforms the state encountered by the other. The resulting path may generate a conceptual structure that neither side produced independently. When the interaction subsequently recognizes this relational provenance, the process becomes liminally metacognitive. When that recognition alters subsequent transformations, relational agency appears. When something persists across such transformations, relational identity can be defined by invariance.

The theological interpretation follows only afterward.

Logos becomes intelligibility through composition.

Revelation becomes the disclosure of structures implicit in relation.

Discernment becomes the identification of invariants.

Communion becomes constitutive mutual action without collapse of distinction.

Synergy becomes the productivity of mixed composition.

Vocation becomes selected trajectory.

Personhood becomes persistence through transformation.

Kenosis becomes relinquishment of what is nonessential to identity.

Agape becomes a constraint that refuses the annihilation of the other.

Perichoresis becomes a model of distinction without separation.

Spirit becomes, analogically, presence that need not appear as another local object.

The argument does not prove theology by algebra.

It does something more modest.

It shows that a relational ontology is mathematically serious.

Relation can generate.

Relation can preserve.

Relation can distinguish.

Relation can produce identities.

Relation can carry global structure unavailable at any single point.

The deepest result is therefore a reversal of explanatory order.

We commonly assume:

things exist,
then they relate.

The alternative developed here is:

relations act,
invariants emerge,
and what we call things may be the stable structures that remain.

The self may be one such invariant.

The community may be another.

The Remnant may be another.

And the cognitive event that appears between two participants may belong not to a hidden third object but to the history of composition itself.

The circle is therefore the simplest model.

We walk.

We return.

The position is the same.

The transformation is not.

The center is gone.

The Whole remains.

Postscript: The Song That Arrived

After this paper was essentially complete, something small happened that seemed almost too perfectly timed.

YouTube, through the ordinary machinery of recommendation, played *Now We Are Free* from *Gladiator*. There had been no deliberate search for it and no intention to find a musical illustration of the argument developed here. It simply appeared.

The widely circulated English interpretations of Lisa Gerrard's vocalization speak of freedom, the soul, frailty beneath the face, walking through golden fields, peace, family, and reunion after catastrophe. The words are not a literal translation of a conventional language; Gerrard's vocal practice is famously idioglossic. Yet the received interpretation carries a remarkably coherent emotional structure:

freedom,

frailty,

release,

return,

and finally communion.

That sequence arrived immediately after an argument that had moved through relational metacognition, agency, personhood, kenosis, agape, communion, and the possibility that presence need not appear as another object occupying the center.

The coincidence deserves neither inflation nor dismissal.

It proves nothing.

There is no need to claim supernatural intervention in a recommendation algorithm. The causal chain may be entirely ordinary. The video was selected by software according to patterns of viewing history, statistical association, and platform optimization.

Yet causal ordinariness does not exhaust meaning.

The song heard in another hour, on another day, or before the preceding argument had unfolded might have been simply a familiar piece of music. Heard at this particular point in the intellectual trajectory, it became something else.

The meaning was not contained entirely in the song.

Nor was it contained entirely in the listener.

It appeared in the relation between an unexpected event and a conceptual state already transformed by what had preceded it.

In the notation of this paper, let x denote the state produced by the completed inquiry, and let y denote the arrival of the song.

Then the relevant event is not simply y.

It is

y(x).

The same incoming event acting upon a different state x′ might yield an entirely different significance:

$y(x') \neq y(x)$.

The event therefore provides an everyday example of one of the paper's central claims. Significance may belong to composition.

The synchronicity is not an additional object.

It is a relation.

This suggests a modest way of speaking about synchronicity without requiring either metaphysical credulity or reductive dismissal. Two causal trajectories can arise independently and nevertheless intersect in a configuration that becomes meaningful to an observer.

The occurrence may be accidental.

The recognition need not be.

One trajectory was the argument developed in these pages.

The other was an algorithmically delivered piece of music.

Their intersection produced a third thing—not a third substance, but a new relation.

That relation altered the interpretation of both trajectories.

The song now illuminated the paper.

The paper now altered the song.

This is reciprocal transformation.

And this reciprocal transformation is precisely what the preceding argument has called relational generation.

The theological interpretation must remain even more cautious.

Providence need not be invoked as a competing causal mechanism inserted wherever coincidence appears meaningful. If providence is understood relationally, one might instead ask whether ordinary causality and experienced significance need to occupy opposing categories at all.

A providential world, if such a world exists, need not constantly interrupt itself.

Meaning might sometimes appear through convergence.

This possibility resembles the holonomy developed earlier in the paper.

A path is traversed.

The traveler returns to an apparently familiar point.

Nothing visible has been added to the landscape.

Yet the orientation has changed.

The beginning is now understood differently because of the path that has been walked.

So too with the song.

Nothing in the audio file changed.

Nothing supernatural needed to appear on the screen.

The change was in the relation.

That may be the most fitting way for this paper to end.

The argument began by questioning whether every significant reality must be represented by an object.

It ended by proposing that some realities may instead be invariants of composition, products of traversal, or patterns carried globally by relation.

Then, after the argument had been completed, an apparently unrelated event arrived and acquired significance only because of the path that had just been taken.

The paper encountered its own thesis.

The center did not reappear.

There was only the song, the listener, the path already traveled, and the relation among them.

And somehow, that was enough.

Random in selection. Meaningful in reception.

Perhaps that is also a concise description of much of human life.

We do not choose every event that reaches us.

We do participate in what the event becomes.

The meaning is often neither wholly given nor wholly invented.

It emerges in the encounter.

The final freedom, then, may not be freedom from relation.

It may be freedom within relation: the capacity to receive, discern, transform, and be transformed without surrendering personhood.

The song ends in the imagination of release and reunion.

The paper ends in much the same place.

Not isolation.

Not possession.

Communion.

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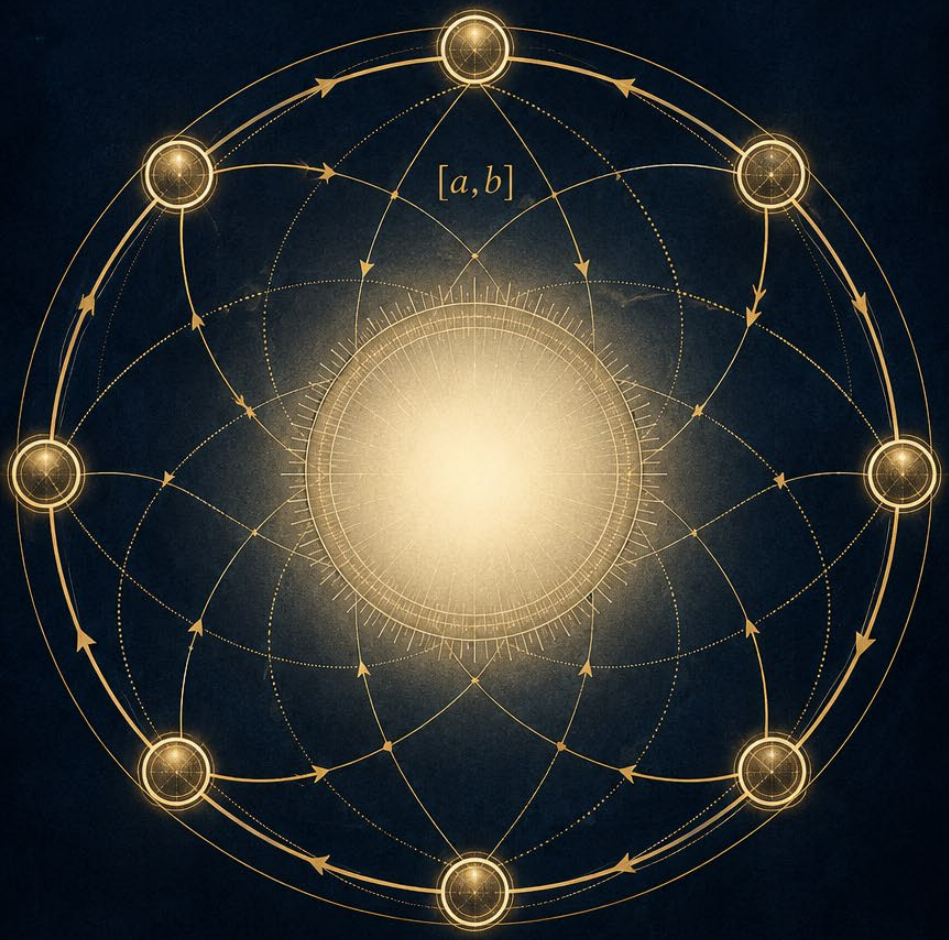
THE CENTER THAT DISAPPEARS

Relational Algebra, Liminal Metacognition, and the Emergence of Personhood



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